BmMT 2025 Online

Team Round

June 7, 2025

Time limit: 60 minutes.

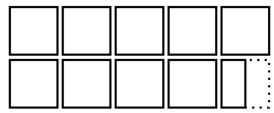
Instructions: For this test, you will work in teams of up to five to solve 20 short answer questions. All answers must be expressed in simplest form unless specified otherwise.

No calculators. Protractors, rulers, and compasses are permitted.

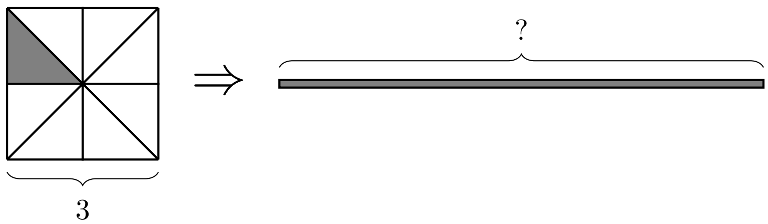
- Carry out any reasonable calculations. For instance, you should evaluate $\frac{1}{2}+\frac{1}{3}$, but you do not need to evaluate large powers such as 7^8.
- Write rational numbers in lowest terms. Decimals are also acceptable, provided they are exact. You may use constants such as π in your answers.
- Move all square factors outside radicals. For example, write $3\sqrt{7}$ instead of $\sqrt{63}$.
- Denominators do *not* need to be rationalized. Both $\frac{\sqrt{2}}{2}$ and $\frac{1}{\sqrt{2}}$ are acceptable.
- Do not express an answer using a repeated sum or product.
- All answers should be written in the form of numbers only. When encountering ratios, express them using integers or fractions and do not use colon notation, for example, use $\frac{3}{4}$ instead of 3:4.
- Do not include any units in your answers.
- For fractions, both improper fractions and mixed numbers are acceptable.

1. What is the value of $16^2 - 11^2 + 6^2 - 1^2$?

2. If Aedan bakes 25 identical brownie pieces, and Brian eats 9 and a half of those pieces, what percentage of the brownies did Brian eat? **If the answer is x%, write only x as your answer.**



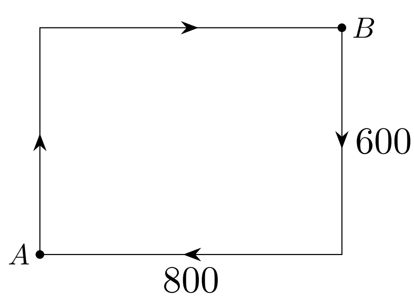
3. A square shaped pizza dough with side length 3 is divided into 8 slices of equal area. One slice is removed and molded (preserving the area) into a long rectangular pizza dough with one side of length $\frac{1}{8}$. What is the length of the longer side?



4. Jeslyn writes down five numbers whose arithmetic average is 6. The first two numbers are 3 and 7, the third number is half the fifth number, and the fourth number is equal to the fifth number. What was the fifth number that Jeslyn wrote down?

5. How many positive even numbers less than 49 are there whose digits sum to an odd number?

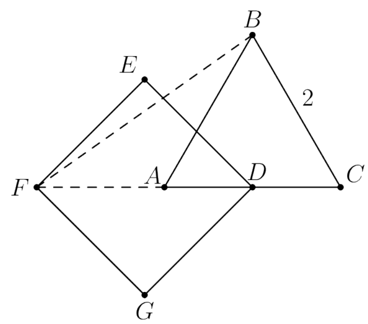
6. Jonathan and Ethan are racing around a rectangular track that is 600 units wide and 800 units long. Jonathan can finish a lap in 4 seconds, while Ethan can finish a lap in 7 seconds. They race one lap around the track, starting at point A and going clockwise. However, once Ethan reaches point B, he cheats by running off of the track, taking the most direct path back to A, at the same speed as before. He still loses the race to Jonathan, who does not cheat. How far away was Ethan from Jonathan when Jonathan finishes the race, *in units*?



7. Isaac picks a number among 1, 2, 3, 4 uniformly at random. Preston picks a number among 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 uniformly at random. What is the probability that Preston picks a strictly larger number than Isaac?

8. Helena writes down the number 0 on a chalkboard. Then, every minute afterwards, she counts how many digits in total are on the board and writes down that number. She repeats this until she has written 36 separate numbers on the board (including the first number, 0). For example, if $0, 3, 12$, and 147 were written on the board, there would be four numbers and seven digits. What is the last number Helena writes?

9. Let triangle $\triangle ABC$ be an equilateral triangle with side length 2, and let D be the midpoint of side $\overline{AC}$, as shown below. If $DEFG$ is a square with side length $DE = BD$ such that A lies on diagonal $\overline{DF}$, what is the value of BF^2?



10. Let x and y be positive integers such that $150x^2y$ and $60xy^2$ are both perfect squares and $3xy$ is a perfect cube. Compute the minimum possible value of xy.

11. Let $\varphi = \frac{1+\sqrt{5}}{2}$. There exist positive integers a and b such that

$$\sqrt{\varphi} + \sqrt{\frac{1}{\varphi}} = \sqrt{a + \sqrt{b}}.$$

Find $a + b$.

12. Aditya chooses a random permutation of the letters that make up "REPOSITORY". What is the probability that Aditya's permutation contains the word "OR" twice? For example, "ORSITYORPE" is one such permutation, but "OROEPSITRY" is not.

13. Call a positive integer n *basic* if there is a positive integer $b > 1$ such that n can be written with b digits in base b (with no leading 0s). For example, 3 is *basic* because it can be written with 2 digits in base 2 as 11_2. How many positive integers $n \leq 2025$ are *basic*?

14. Distinct points A, B, T, and D lie on a line such that $AB = BT = TD = 40$. Points E and F satisfy $AE = DF = 48$ and $TE = BF = 64$, with line segments $\overline{TE}$ and $\overline{BF}$ intersecting at a point M. What is the perimeter of triangle $\triangle BMT$?

15. For integers a and b, define the binary operation $\bigstar$ by

$$a \bigstar b = a + b + ab.$$

There exists an associative operation $\blacktriangledown$, meaning that $(a \blacktriangledown b) \blacktriangledown c = a \blacktriangledown (b \blacktriangledown c)$ for all real numbers a, b and c, such that whenever x is a non-negative integer,

$$x \bigstar y = \underbrace{y \blacktriangledown \cdots \blacktriangledown y \blacktriangledown y}_{x\ \blacktriangledown\text{'s}}.$$

Compute $6 \blacktriangledown 7$.

16. Square $ABCD$ has side length $4 + 2\sqrt{2}$. A circle is drawn such that it passes through C and is tangent to sides $\overline{AB}$ and $\overline{AD}$. The area of the total region covered by at least one of the circle and the square can be written as $a + b\sqrt{c} + d\pi$, where a, b, c, and d are all integers, and c is square-free (it is not divisible by any perfect square greater than 1). Find $a + b + c + d$.

17. What is the least positive number of zeroes that can be concatenated to the end of 2025 such that the sum of the even factors of the resulting number is divisible by 45?

    Here "concatenated" means writing zeroes at the end of the number.

    For example: Concatenating one zero to "2025" gives "20250", concatenating two zeroes gives "202500", and so on.

18. What is the greatest possible value of C satisfying the property that the following system of equations
$$x^2 = y + C, \quad y^2 = x + C$$
has exactly 2 real solutions, and all solutions are real?

19. Consider a bee (denoted by X) in a rectangular honeycomb as seen below:

	1	2	3	4	5	6	7
A				X			
B							
C							
D							

    In one move, the bee may move to an adjacent square via an up, down, left, or right move, and it can no longer move once it reaches row D. The bee cannot move outside the honeycomb. It cannot revisit a square it has already been to, and it cannot move more than six times. Find the number of different paths the bee can take from its starting point to row D.

20. Let ω_1 be a circle with center O and radius 4 and ω_2 be a circle with center P and radius 1 such that ω_1 and ω_2 are externally tangent to each other and both tangent to line $\overleftrightarrow{AB}$, with ω_1 tangent at A, and ω_2 tangent at B. Point D lies on ω_2 such that O, P, and D are collinear and D is not on ω_1. Line ℓ is tangent to circle ω_2 at D. Let C be the point of intersection of line $\overleftrightarrow{OA}$ and line ℓ, and let K be the point of intersection of line $\overleftrightarrow{AB}$ and line ℓ. If Q is a point inside triangle $\triangle AKC$ such that triangle $\triangle OQD$ is larger than $\triangle AQC$, what is the area of the region of possible locations of Q?